FREE DISTRIBUTIVE SEMIGROUP

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ABSTRACT

The paper [1] represents the construction of free medial semigroup through commutative semigroup. In this project the structure of distributive semigroup [2], [3], [4] and the commutative semigroup are been used for constructing one distributive semigroup through commutative semigroup. The first part explains the distributive semigroups and commutative semigroups, represents the conditions that the commutative semigroup should complete whereof to construct distributive semigroup. The second part contains the contruction of the free distributive semigroup using indications of the first part. Pointing that in this project I will try the same technique to accompany the different semigroups.

Key words. *Distributive semigroup, Commutative semigroup, homomorphism.*

0.Introduction

A semigroup S is called left(right) distributive semigroup if $xyz =$ $(yzx =$ where x, y, z belong to S.

A semigroup S is distributive semigroup if it is both left and right distributive semigroup.

S is a commutative semigroup if it satisfies $xz =$, for any $x, z \in$.

Definition: $x \in$ is an idempotent if $x^2 =$ .Also we define the set of idempotents in S to be

$Id(S) =$ $\in$ $=$

Now, $Id(S)$ may be empty, but $Id(S)$ may also be S.

Let X be a finite alphabet, $F[X]$ denote the set of nonempty words $a_1 a_2 ... a_n$ over X

The binary operation "· an $F[X]$ is defined

(*) …….. $a_1 a_2 ... a_n$ $b_1 b_2 ... b_n$ $=$ $b_1 b_2 ... b_n$

The set $F[X]$ with a binary multiplicative operation defined by (*) is called free semigroup over X

If $A =$ $., a_n$ is finite and if the congruence ρ on $F[X]$ is generated by a finite set

$R =$ $\in$ $\times$

then we said $F[X]/\rho$ is constructed with a finite number of generators, so

$$F[X]/\rho =< \quad > \; = \quad = \; >$$

$F[X]/\rho$ have the system of free generators $a_1, a_2, ..., a_n$ and the relations $w_1 =$ $=$

1.Free distributive semigroup

Let $(S,+$ be a commutative semigroup and $\varphi\ \psi$ its idempotent permutable endomorphisms:

(1) … φ $=$ $=$ $=$ and $\varphi\psi$ $=$ for all $x, y \in$.

Proposition1: Let $(S,+$ be a commutative semigroup of idempotents and φ,ψ its endomorphisms satisfying (1). The operation "$\cdot$" is defined as follows

(2)... $xy = \quad + $

Then, $(S,.)$ is a distributive semigroup.

Proof: Let $x,y,z \in$, then

$xyz = \quad + \quad = \quad + \quad + \quad =$

$= \quad + \quad + \quad = \quad + \quad + \quad + \quad =$

$= \quad + \quad =$

$yzx = \quad + \quad = \quad + \quad + \quad =$

$= \quad + \quad + \quad =$

$= \quad + \quad + \quad + \quad =$

$= \quad + \quad =$

$=$

We proof that $(S,.)$ is a distributive semigroup.

2.Construction

Now we will construct free distributive semigroups with system of free generators $x_i \ ,i \in$

Let $(F,+$ be free commutative semigroup of idempotent with system of free generators

$a_i \cup \quad \cup \quad \cup \quad a_i \cup \quad \cup \quad \cup \quad i \in$

Now on $(F,+$ we define the homomorphisms φ and ψ as follows:

(3) …….. $\varphi = \left(\quad \right)$ and $\psi = \left(\quad \right)$

Proposition 2: Homomorphisms φ and ψ of free idempotent commutative semigroup $(F,+$ defined by 3 satisfies 1 and $F,.$ where "$\cdot$ is defined by 2 is distributive semigroup.

Proof: Homomorphisms φ and ψ satisfies (1) because this conditions satisfied on system of free generators. By the proposition1 $(F,\cdot$ is a distributive semigroup. #

Proposition 3: Let a_i , $i \in$ be a system of free generators. Then $F \ a_i, i \in \quad \cdot$, is a free distributive semigroup with a system of free generators a_i , $i \in$.

The elements from $F \ a_i, i \in$, have the form

$(4a)$ $w_1 = \prod_{k=}$ where $a_{i_\nu} \neq \quad_\mu$ for $\nu \neq$, $\nu \ \mu = \{2,3,...,r- \}$ and $n_1, n_r \in$

or

$(4b)$ $w_1 = \prod_{k=}$ where $a_{i_\nu} \neq \quad_\mu$ for $\nu \neq$.

Let:

$(5a)$ $w_2 = \prod_{l=}$ or

(5b) $w_2 = \prod_{l=}$

From $w_1 = \Rightarrow r =$ and $a_{i_1} = =$ such that the ecuations (a) or (b) take the form:

(6) $\prod_{k=}$ and $\prod_{l=}$.

From $w_1 \neq$ and $w_2 \neq$ we have

$w_1 = + + = + +$ and

$w_2 = + + = + +$

From $w_1 = \Rightarrow i_1 =$ and $i_r =$, so from 6 $\Rightarrow a_{i_r} =$, and $r =$, $n_1 =$, $n_r =$.#

It is obvious that for the construction of free distributive semigroups $F\ a_i, i \in$ $\cdot$ it is not taken minimal commutative idempotent semigroup $(F, +$.

Let $(F, +$ be free commutative idempotent semigroup with a system of free generators $a \cup \cup \cup$.

Let φ and ψ be homomorphisms of $(F, +$ defined as follows:

$\varphi = \begin{pmatrix} \\ \end{pmatrix}$ and $\psi = \begin{pmatrix} \\ \end{pmatrix}$

Homomorphisms φ and ψ satisfy 1 .

Proposition4: $(F$ [a,c]$, \cdot$ is a free distributive semigroup with a system of free generators $a \cup$.

Proof: All elements of $F[a,c]$ have the form $a^m, c^n, a^m c^n, c^n a^m, a^m c^n$ and $c^n a^m c$ for $1 \leq \leq$.

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