MATHEMATICAL METHOD FOR CALCULATING ON THE TEMPERATURE IN A FLAT PLATE SOLAR COLLECTOR

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ABSTRACT

In this paper is is offered mathematical method for calculating of the temperature in a flat plate solar collector. In this collector is done converting solar radiation into heat. The temperature in the plastic container is measured by a thermocouple electronic thermometer until a constant temperature (steady state).

Newton's interpolation formulae for unevenly spaced values of an argument are used as a mathematical method to compute the temperature. Newton's interpolation polynomial is constructed for 20 interpolation angles at which the polynomial value is the same as the measured temperatures. This polynomial makes it possible to determine the temperature different from the interpolation angles. It is made assessment of the accuracy. There is a good coincidence of experiment and theory.

Key words: solar collector, Newton's interpolation formulae, interpolation angles, mathematical methods for analysis and an estimate.

INTRODUCTION

The solar thermal collector is a device, which performs conversion of solar radiation into heat that accumulates in this device. The principal scheme of a flat solar collector is given in Fig. 1 [1, 2,10].

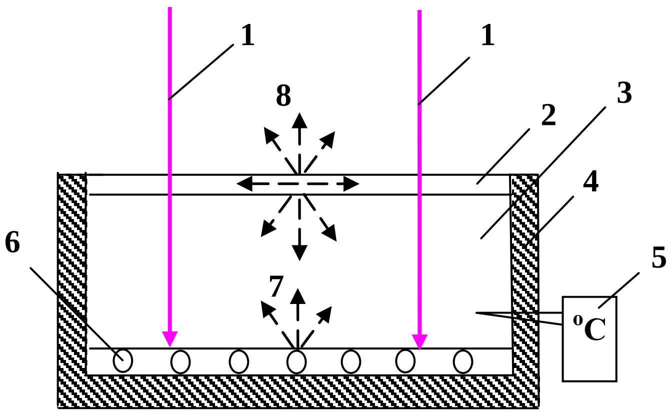

Fig. 1. The principal scheme of a flat solar collector.

In Fig. 1: 1 - sun rays falling perpendicular to the transparent cover (lid) 2; 3 - air; 4 - insulation; 5 - thermocouple; 6 - pipes with a heat carrier; 7 - radiation from the absorber; 8 - radiation from the cover. The glass and foil coatings serve to reduce the loss of heat radiation and those of air convection with surrounding atmosphere.

The main part of the solar thermal collector is the absorber, which is a metal plate with a dark absorbent surface. In some cases, it is covered with a special selective coating, which is characterized by high absorption capacity with respect to solar radiation. The absorber has properties similar to the properties of an ideal black body - a term used in the theory of thermal radiation (model body that completely absorbs all radiation that falls on it) [7, 8, 9].

The cover (of glass or polymer film) is relatively transparent to visible light ($400 < \lambda < 800$ nm) and that of the near infrared area ($800 < \lambda < 4000$ nm), that is about 75% of the total solar radiation [5]. By law Vin from physics, you can calculate the maximum length λ_{max}, that emits the absorber. At room temperature, the emission maximum of the ideal black body is in the far infrared range (approximately λ_{max} = 10000 nm).

The penetrated solar radiation are absorbed and accumulated into the collector. As can be seen from Fig. 1, the heat rays from the bottom of the collector reached cover 2 are absorbed by it. About half of those absorbed rays are emitted (lost) out of the collector and the other half "recycled" back again to the absorber. This leads to the amplification of warming, known as greenhouse effect [4]. If heat is not removed from the manifold, the temperature may exceed 150 ° C.

Newton's interpolation formulae for unevenly spaced values of an argument are used as a mathematical method to compute the temperature from the computing mathematical methods of analysis [3,6]. Newton's interpolation polynomial is constructed for 20 interpolation angles at which the polynomial value is the same as the measured temperatures. This polynomial makes it possible to determine the temperature different from the interpolation angles. It is made assessment of the accuracy. There is a good coincidence of experiment and theory.

METHODS OF STUDY

Newton's interpolation formulae for unevenly spaced values of an argument are used as a mathematical method to compute the temperature.

In the interval [x_o, x_n], n + 1 points $x_o, x_1, \ldots, x_n$, which are the interpolation angles, have been assigned. In our case, n + 1 = 21. Hence, Newton's interpolation formula for unevenly spaced values of an argument has the following general form:

$$P(x) = y_o + [x_o, x_1].(x-x_o) + [x_o, x_1, x_2].(x-x_o).(x-x_1) + \\ + \ \ldots \ + [x_o, x_1, \ldots, x_n].(x-x_o).(x-x_1) \ldots (x-x_{n-1}) , \qquad (1)$$

where: y_o is the initial value of the function; [x_o, x_1] are first order differential values:

$$[x_o, x_1] = \frac{y_1 - y_o}{x_1 - x_o} \ ;$$

[x_o, x_1, x_2] are second order differential values:

$$[x_o, x_1, x_2] = \frac{[x_1, x_2] - [x_o, x_1]}{x_2 - x_o} \ ;$$

... [x_o, x_1, ... , x_n] are nth order differential values:

$$[x_o, x_1, \ldots, x_n] = \frac{[x_1, \ldots, x_n] - [x_o, \ldots, x_{n-1}]}{x_n - x_o} .$$

Polynomial (1) is represented in the following form:

$$P(x) = y_o + \sum_{i=1}^{20} A_i \ , \qquad (2)$$

where A_i = [$x_o, x_1, \ldots, x_i$].$x(x - x_1)(x - x_2)\ldots(x - x_{i-1})$.

The values of A_i for the assigned interpolation angles $x_o, x_1, \ldots, x_{20}$ have been calculated. The temperatures determined by Newton's interpolation angle polynomial coincides with is the same as the measured temperatures, as should be the case.

Newton's interpolation polynomial proposed here (2) can be used to calculate the value of the function for points different from the interpolation angles in the [x_o, x_n] interval.

RESULTS AND DISCUSSION

A model of a flat plate solar collector (Fig. 1) was made from suitable light plastic material, which was thermally insulated with Styrofoam 4 or Fibran and tanned on the inside. The beams 1 of the light stimulator hit a transparent covering (made of glass and plastic folio) perpendicularly. The temperature inside the greenhouse was measured with a pre-calibrated differential thermocouple 5.

The experiment was carried out with empty plastic vessel (air) with a volume of 400 cm^3 and the light source was a 200 W electric lamp. From the measured values of temperature (Table 1) shows that after a while it varies more slowly up to the steady state (316.15 K).

Table 1. The measured values of temperature.

t [min]	*t* [°C]	*T* [K]	*t* [min]	*t* [°C]	*T* [K]	*t* [min]	*t* [°C]	*T* [K]	*t* [min]	*t* [°C]	*T* [K]
0	24	297.15	15	31	304.15	45	37	310.15	100	42	315.15
2	26	299.15	20	32	305.15	50	38	311.15	110	42.2	315.35
4	27	300.15	25	33.5	306.65	60	39	312.15	120	42.6	315.75
6	28	301.15	30	35	308.15	70	40	313.15	130	43	316.15
8	29	302.15	35	35.5	308.65	80	40.5	313.65	140	43	316.15
10	29.8	302.95	40	36	309.15	90	41	314.15	160	43	316.15

Newton's interpolation formulae for unevenly spaced values of an argument are used as a mathematical method to compute the temperature. Newton's interpolation polynomial is constructed for 20 interpolation angles at which the polynomial value is the same as the measured temperatures.

The computed differential values are presented in table 2. The values of A_i for the assigned interpolation angles x_o, x_1,…, x_{20} have been calculated. The temperature obtained using the proposed polynomial for the interpolation angles coincides with the measured values of temperature, as should be the case.

With compositions the Newton's interpolation polynomial (2) an be calculated from the value of the feature points other than the corners of the interpolation in the range [0,160]. The calculations are made for values of time which are of our experiments, but are different from the angles of interpolation. These are the points 10, 35 and 50. For them it is been evaluated the accuracy.

The absolute error is the difference between the temperature obtained in the experiment T_{EXP} and the calculated temperature T_{CALC} with Newton's interpolation polynomial for intermediate values of the angles of interpolation:

$$\Delta T = T_{EXP} - T_{ИЗЧ} \quad (3)$$

The relative error of the result is:

$$\varepsilon\% = \frac{\Delta T}{T}\% = \frac{T_{EXP} - T_{CALC}}{T_{EXP}}. \quad (4)$$

In Fig. 2 is presented graphically change the temperature inside the collector. Derived is the regression equation of the type:

$$y = ax^2 + bx + c, \quad (5)$$

where: y - temperature ^{0}C; x - time, s; a, b and c - regression coefficients. The Coefficients have values:

a = -0.001113; b = 0.273053; c = 26.484257.

The quality of the regression equation is evaluated by the coefficient of determination R^2. Fig. 2 shows that a value R^2 very close to the unit (R^2= 0.979326), indicating that the equations (5) can be used to calculate the temperature inside the collector. This method is widely used in research.

By the Newton's interpolation formulae can also calculate the temperature for points other than the angles of interpolation. As can be seen from Table 3, the relative errors with polynomials of Newton have smaller values (eg. for t = 10 min ε% = 0.079%) than that calculated by the regression equation (e. For t = 10 min ε% = 2.393), indicating that Newton's method is suitable for analysis and prediction.

Table 2. Calculated divided differences by Newton's interpolation method.

	x	*y*	I order	II order	III order	IV order	V order	VI order	VII order
x_o	0	24	1	-0.125	0.02083	-0.0026	0.000163	-7.1E-06	2.3E-07
x_1	2	26	0.5	0	0	-0.0002	2.1E-05	-1.4E-06	5.9E-08
x_2	4	27	0.5	0	-0.0022	0.00021	-1E-05	2.95E-07	-4E-09
x_3	6	28	0.5	-0.02381	0.00119	-1E-05	-3E-06	1.52E-07	-4E-09
x_4	8	29	0.29	-0.00714	0.00101	-8E-05	2.4E-06	-4.2E-09	-2E-09
x_5	15	31	0.2	0.00667	-0.0007	0	2.2E-06	-1.1E-07	3.3E-09
x_6	20	32	0.3	0	-0.0007	6.7E-05	-3E-06	7.38E-08	-2E-09
x_7	25	33.5	0.3	-0.0133	0.001	-4E-05	1.1E-06	-2.4E-08	4.6E-10
x_8	30	35	0.1	0.00667	-0.0003	0.00001	-3E-07	5,86E-09	-1E-10
x_9	40	36	0.2	-0.0033	6.7E-05	-3E-06	1E-07	-2.5E-09	1.3E-11
x_{10}	45	37	0.13	-0.0013	-3E-05	2.6E-06	-5E-08	-1.6E-09	1.3E-10
x_{11}	60	39	0.1	-0.0025	8.3E-05	0	-2E-07	8.19E-09	-3E-10
x_{12}	70	40	0.05	0	8.3E-05	-7E-06	3.4E-07	-1.1E-08	2.4E-10
x_{13}	80	40.5	0.05	0.0025	-0.0002	9.6E-06	-3E-07	6.25E-09	-8E-11
x_{14}	90	41	0.1	-0.004	0.00017	-5E-06	8.3E-08	-4.4E-10	
x_{15}	100	42	0.02	0.001	-3E-05	-8E-07	5.3E-08		
x_{16}	110	42.2	0.04	0	-7E-05	2.3E-06			
x_{17}	120	42.6	0.04	-0.002	5E-05				
x_{18}	130	43	0	0					
x_{19}	140	43	0						
X_{20}	160	43							

VIII order	IX order	X order	XI order	XII order	XIII order	XIV order	XV order	XVI order
-6E-09	1E-10	-1E-12	1.2E-14	-3E-17	-1E-18	3.3E-20	-5E-22	7.2E-24
-2E-09	3.9E-11	-7E-13	9.7E-15	-1E-16	1.7E-18	-2E-20	2.5E-22	-3E-24
-3E-13	6.7E-13	1.2E-15	-5E-16	1.8E-17	-4E-19	6.2E-21	-8E-23	9.7E-25
3.7E-11	7.5E-13	-4E-14	1E-15	-2E-17	2.7E-19	-3E-21	3.8E-23	-4E-25
8.5E-11	-2E-12	4.4E-14	-7E-16	1E-17	-1E-19	1.3E-21	-1E-23	9.8E-26
-8E-11	1.4E-12	-2E-14	3E-16	-4E-18	3.8E-20	-3E-22	2.4E-24	
3E-11	-5E-13	6.4E-15	-8E-17	7.4E-19	-5E-21	4.5E-24		
-8E-12	1.1E-13	-1E-15	4.9E-18	1.6E-19	-4E-21			
1.7E-12	-2E-15	-7E-16	2.3E-17	-4E-19				
1.5E-12	-7E-14	1.8E-15	-3E-17					
-5E-12	1.2E-13	-2E-15						
6.3E-12	-1E-13							
-4E-12								

XVII order	XVIII order	XIX order	XX order
-8E-26	8.7E-28	-8.3E-30	6.7E-32
3E-26	-3E-28	2.35E-30	
-1E-26	8.3E-29		
3.1E-27			

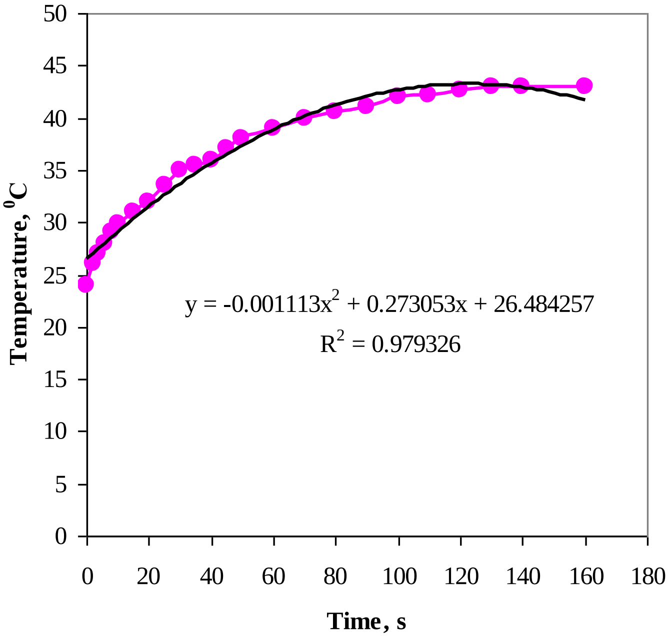

Fig. 2. Graph of the change in temperature with the time.

Table 3. Calculated values T_{CALC} and ε%, computed by the Newton's interpolation polynomial (2) and the regression equation (5).

t [min]	T_{EXP} [°C]	Newton's interpolation polynomial		regression equation	
		T_{CALC} [°C]	ε%	T_{CALC} [°C]	ε%
10	29.8	29.8235	0.079	29.1035	2.393
35	35.5	35.7400	0.672	34.6778	2.371
50	38	38.8412	2.166	37.3544	1.728

The method can be used to study the change of the temperature of the solar collector. Such information would be useful for practice (design of solar systems, greenhouses, photovoltaic and others.).

CONCLUSIONS

1. In this paper is is offered mathematical method for calculating of the temperature in a flat plate solar collector. Newton's interpolation formulae for unevenly spaced values of an argument are used as a mathematical method to compute the temperature.

2. With compositions the Newton's interpolation polynomial (2) an be calculated from the value of the feature points other than the corners of the interpolation in the range [0,160]. The calculations are made for values of time which are of our experiments, but are different from the angles of interpolation. These are the points 10, 35 and 50.

3. Derived is the regression equation. The relative errors with polynomials of Newton have smaller values (eg. for t = 10 min ε% = 0.079%) than that calculated by the regression equation (e. For t = 10 min ε% = 2.393), indicating that Newton's method is suitable for analysis and prediction.

4. By the Newton's interpolation formulae can also calculate the temperature for points other than the angles of interpolation. As can be seen from Table 3, the relative errors with polynomials

of Newton have smaller values (eg. for t = 10 min ε% = 0.079%) than that calculated by the regression equation (eg. for t = 10 min ε% = 2.393), indicating that Newton's method is suitable for analysis and prediction.

5. The method can be used to study the change of the temperature of the solar collector. Such information would be useful for practice (design of solar systems, greenhouses, photovoltaic and others.).

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