

INTERQUARK POTENTIAL WITH QCDCALAB2 IN LATTICE QCD

Dafina Hyka (Xhako)*, Rudina Osmanaj (Zeqirllari)**

*Polytechnic University of Tirana, Department of Physics Engineering, Boulevard "Dëshmorët e Kombit", Square "Mother Teresa", 4. 1000 Tirana, Albania

**University of Tirana, Faculty of Natural Sciences, Department of Physic, Tirana, Albania.

Corresponding Author: dafinaxhako@yahoo.com

ABSTRACT

The calculation techniques of lattice QCD (Quantum Chromodynamics) offer special opportunities to study nonperturbative effects of strong interaction in physics. In lattice QCD the static interquark potential can be derived from the Wilson loops. In this paper we use a special package named QCDCALAB2. We use planar Wilson loops to determine the static interquark potential. Using simulation with SU(3) gauge field configuration we derive quark-anti-quark potential for different values of coupling constant and for different lattice sizes. The calculations are made for 100 configurations, statistically independent, of gauge fields of the lattice.

Key-words: *interquark potential, lattice constant, QCDCALAB2, SU(3) gauge theory*

INTRODUCTION

Although Quantum Chromodynamic (QCD) is the best candidate for a theory of the strong interactions, the quantitative information that can be extracted is still limited. Analytic or perturbative solutions in low-energy QCD are hard or impossible due to the highly nonlinear nature of the strong force.

The lattice formulation (Lattice-QCD) [1] is one of the most elegant and powerful non-perturbative methods. It is a lattice gauge theory formulated on a grid or lattice of points in space and time. Where, fields representing quarks are defined at lattice sites while the gluon fields are defined on the links connecting neighboring sites. This formulation of QCD in discrete rather than continuous space-time naturally introduces a momentum cut off at the order $1/a$, where a is the lattice spacing (lattice constant), which regularizes the theory. This approximation approaches continuum QCD as the spacing between lattice sites is reduced to zero. As a result, lattice QCD is mathematically well-defined. Its power comes mainly from the possibility to evaluate physical quantities "exactly" via numerical simulations, yielding a method to test whether QCD provides the correct framework for describing strong interactions. The lattice spacing plays the role of the ultraviolet regulator, rendering the quantum field theory finite. Lattice QCD has already made successful contact with many experiments. For example, the mass of the proton has been determined theoretically with an error of less than 2 percent.

An object of considerable theoretical interest that has been studied for almost 30 years is the potential between a static quark and anti-quark (the Bosonic string, see Appendix I). It defines the energy of gauge fields in the presence of two static color sources separated by a distance R . At large distances quark confinement shows up and perturbative methods are no longer able to describe the behavior of physical observables; in this context, lattice gauge theories play an important role. Lattice results can be used to investigate non-perturbative features of the static quark potential as well as to test the range of validity of perturbation theory, which is still an important phenomenological issue which has to be investigated more precisely. The Creutz computer experiment in 1970 [2] to measure the static quark-anti-quark potential in essence have been that quarks are *confined*, so that the

static potential exhibits a linear rise at large distance. Recent lattice studies [3, 4, 5, 6] confirm the existence of the linear potential for the intermediate distances for the fundamental and higher representations in SU(3). Based on these measurements, quarks are confined in all representations of SU(3).

Motivated by these important physical results we measure the static quark-anti-quark potential in quenched approximation using QCDCALAB2 software [7].

MATERIAL AND METHODS

A Wilson loops were introduced in the 1970s in an attempt at a non-perturbative formulation of quantum chromodynamics (QCD), or at least as a convenient collection of variables for dealing with the strongly-interacting regime of QCD. The use of Wilson loops to measure the string tension in non-abelian gauge theories is the simplest framework in which non-perturbative quantities can be computed (at least numerically) with good precision. Wilson ideas [8] find here a practical implementation and become operative tools. The problem of confinement, which Wilson loops were designed to solve, remains unsolved to this day. The measurements on the lattice of the string tension and of the potential, both related to Wilson loop measures, have parallel histories initiated for the potential in [9, 10, 11, 12, 13].

We have used the standard methods to compute or “measure” the quark-anti-quark potential. The quark-anti-quark potential can be extracted by large time behavior of the Wilson loops. We start from a closed rectangular path $C(R, T)$ with extension $R \times T$. The potential is extracted from so called Wilson loops $W(R, T)$, which are defined to be the trace of path-ordered products of link variables $U_\mu(n)$ along the path $C(R, T)$. This loop construction corresponds to the world lines of a quark-anti-quark pair at rest, separated by distance R from each other and “travelling” over time separation T . In Euclidian time, this observable will reveal the static “ground state” energy for large T values. To calculate the energy, one must compute the correlator function of quark anti-quark operator for different time:

$$W(R, T) = \langle 0 | O_R(0) O_R(T)^* | 0 \rangle = \langle O_R(0) O_R(T)^* \rangle \quad (1)$$

when operator $O_R(t)$ is gauge invariant and is defined as:

$$O_R(t) = \bar{q}(t, 0) U((t, 0) \rightarrow (t, R)) q(t, R) \quad (2)$$

and $U((t, 0) \rightarrow (t, R))$ is gauge field that connect static quarks from point $(t, 0)$ to point (t, R) . Finally, the Wilson loops can be defined as:

$$W(R, T) = \langle \text{tr} U((0, 0) \rightarrow (0, R)) U((0, R) \rightarrow (T, R)) U((T, R) \rightarrow (T, 0)) U((T, 0) \rightarrow (0, 0))^* \rangle \quad (3)$$

The Wilson loops can be written as:

$$W(R, T) = \sum_{n \geq 1} c_n e^{-V_n(R)T} \quad (4)$$

Where the ground state $V_1(R) \equiv V(R)$ and other values for $n>1$ are the potentials of the excited states. So, we can extract potential assuming:

$$W(R, T) \cong c_1 e^{-V(R)T}, \quad (5)$$

and finally calculate the effective potentials from:

$$V(R, T)_{eff} = -\log \frac{W(R, T+1)}{W(R, T)} \quad (6)$$

Finally, for different R we select as $V(R)$ the median value of $V(R, T)_{eff}$ over all T sizes of Wilson loops, and the error is computed using the jackknife method (see Appendix II). These data are fitted to the general form in physical unit:

$$V(R, T)_{eff} = V_0 + KR + \frac{\alpha}{R} \quad (7)$$

where α is a constant (the coefficient of the Coulomb-like term) and K the string tension parameter. We can take the representation of this model in lattice unit by multiplying with lattice parameter the equation (7):

$$\hat{V}(R)_{eff} = \hat{V}_0 + \hat{K} \frac{R}{a} + \frac{a}{R} \hat{\alpha}. \quad (8)$$

For our calculation we have used as environment QCDLAB2 program, which is a small set of short programs that allows to study the basic properties of QCD without necessary of advanced computing technology and associated algorithms. In contrast to other software, QCDLAB2 maps linear operators of QCD to linear operators of the GNU Octave language. Although GNU Octave is an interpreted language, linear operators are precompiled. This property enables very efficient coding as well as minimal run times.

RESULTS AND DISCUSSION

The simulation are made with Wilson action on $12^3 \times 32$ lattice volume at coupling constant 5.7, for a total number of 100 SU(3) gauge configurations using QCDLAB2 software. We have generate rectangles Wilsoon loops with $R_{max} = T_{max} = 4$. We have calculated the lattice spacing a and as well its error. The lattice spacing, which set the scale of the theory, resulted to be $a = 0.22(02)$ with error: $err(a) = 0.03(46)$. From the 3-sigma band plot (Figure 1) we see that the potential is of the order of 1 GeV if it is extrapolated at 1 fm(fermi) separation.

As we can see from graphic of interquark potential, we find that the quark-anti-quark potential is dominated by a Coulomb-like term at short distances with an R dependent coupling $V(R) \cong V_0 + \alpha/R$ for R small and by a linearly rising term at large distances $V(R) \cong V_0 + KR$ for R large, that confirm the fact of confinement of quarks within hadrons.

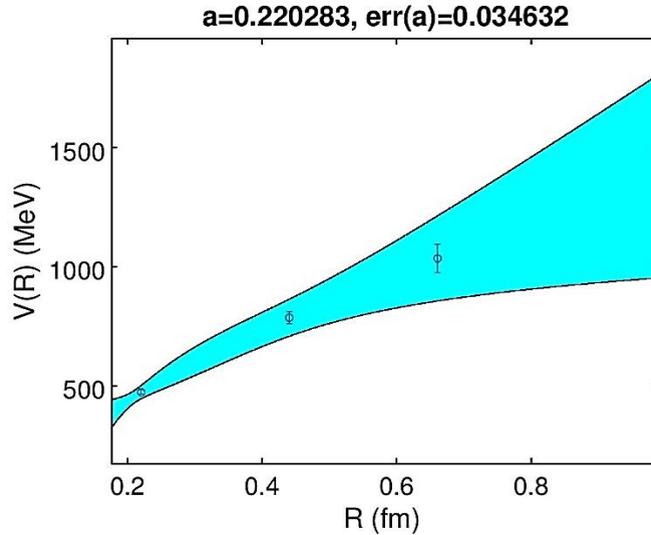


Figure 1: Interquark potential using rectangles Wilson loops on $12^3 \times 32$ lattice volume at coupling constant 5.7 with QCDLAB2 software. In the figure it is shown also the 3-sigma band error.

This expected result confirm the effectiveness of QCDLAB2 software as the right environment to develop in the future algorithms for simulation of lattice QCD theory in SU(3) gauge field. The results of calculation of interquark potential with QCDLAB2 confirms that quark are confined into hadrons. QCDLAB2 is a set of codes very efficient in lattice QCD simulations in SU(3) gauge field. Motivated by results of this work, in our future work we have to develop inversion algorithms for chiral fermions in SU(3) gauge theory using as basic software, QCDLAB2

Appendix I

The model of Bosonic string

One of the main goals of a non-perturbative formulation of Gauge Theories is trying to understand the phenomenon of quark confinement. In a non-Abelian gauge theory color sources are confined in color-singlets, which cannot be separated. At a qualitative level such confinement is understood by assuming that the color electric fluxes emanating from the quarks are squeezed into a string-like configuration (figure 2). Such a gauge field configuration has a constant energy density per unit length, i.e. $E = KR$. This energy density K is called the string tension, and it is the fundamental physical quantity for a pure gauge field. According to this model the quarks are supposed to be connected with “strings” which represent flux color of quarks (Nambu 1974).

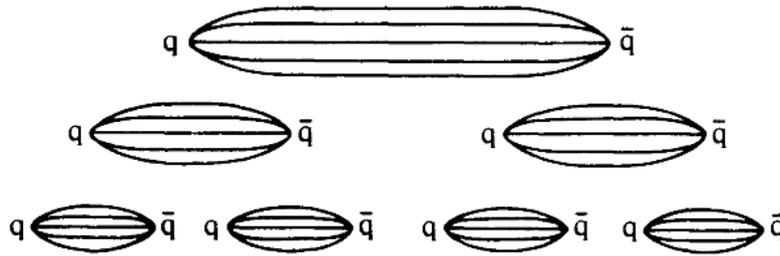


Figure 2: The schematic representation of quark-anti-quark flux color tubes.

Production of a quark-anti-quark pair by vacuum or by annihilations of pair e^+e^- leads to the cutting of string as its length R increase. This makes the initial string to create two new strings and so on. This process stops until the length of strings will be of hadronic scale i.e. 1fm (200MeV-1), or until the energy will be of hadron masses range i.e $\sim 1\text{GeV}$.

Appendix II

Jackknife method to calculate statistical errors

It is no problem to calculate the expectation values and statistical error estimates of “normal” observables from Monte Carlo. However, often we have to calculate functions which depend (possibly non-linearly) on the expectation values of some quantities. Jackknife method is nowadays standard way to calculate the error in this case. Jackknife estimators allow to correct for a bias and its statistical error. The method was introduced in the 1950s by Quenouille and Tukey. The jackknife method is recommended as the standard for error bar calculations. In unbiased situations the jackknife and the usual error bars agree. Otherwise the jackknife estimates are improvements, so that one cannot loose. The jackknife estimation of a parameter is an iterative process. First the parameter is estimated from the whole sample. Then each element is, in turn, dropped from the sample and the parameter of interest is estimated from this smaller sample. This estimation is called a partial estimate (or also a jackknife replication). A pseudo-value is then computed as the difference between the whole sample estimate and the partial estimate. These pseudo-values reduce the (linear) bias of the partial estimate (because the bias is eliminated by the subtraction between the two estimates). The pseudo-values are then used in lieu of the original values to estimate the parameter of interest and their standard deviation is used to estimate the parameter standard error which can then be used for computing confidence intervals. Below we will list the main steps that follow this method:

Step 1: Create new random samples X_i for $i = 1, 2, \dots, n$ from data samples x_i for $i = 1, 2, \dots, n$ according to:

$$X_i = \bar{x} - \frac{x_i - \bar{x}}{n - 1}$$

Science & Technologies

Step 2: Calculate the derive quantity in function of new samples from:

$$y_i = f_i(X_1, \dots, X_n)$$

Step 3: Define the statistical error of \bar{y} according to relation:

$$s_{\bar{y}} = \frac{n-1}{n} \hat{\sigma}_i^2 (y_i - \bar{y})^2$$

BIBLIOGRAPHY

-
- [1] C. Gattringer, C. B. Lang, "Quantum Chromodynamics on the Lattice" Springer , ISBN: 3642018491 ,343 pages (2009).
 - [2] K. M. Creutz and K.J.M. Moriarty, Phys. Rev. D26, 2166-2168 (1982).
 - [3] K. Schilling, G.S. Bali Int.J. Mod.Phys.C4:1167-1177, (1993)
 - [4] S. Deldar, Phys. Rev. D62, 34509 (2000).
 - [5] G. Bali, Phys. Rev. D62, 114503 (2000).
 - [6] G. Parisi, R. Petronzio, and F. Rapuano. Phys.Lett., B128:418, (1983)
 - [7] <https://sites.google.com/site/artanborici/qcdlab>
 - [8] K. G. Wilson, Phys. Rev. D10, 2445 (1974)
 - [9] S. Aoki et al., CP-PACS collaboration, Nucl. Phys. Proc. Suppl. 63 161, (1998).
 - [10] M. Luscher, R. Sommer, P. Weisz and U. Wolff, Nucl. Phys. B413 481, (1994)
 - [11] S. Capitani et al., Nucl. Phys. Proc. Suppl. 63 153, (1998).
 - [12] S R.G. Edwards, U.M. Heller and T.R. Klassen, hep-lat/9711003, (1997).
 - [13] R. Sommer, Nucl. Phys. B411 839, (1994).